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LETTER TO THE EDITOR

On a bosonization approach to disordered systems

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Abstract
In [1] a new bosonization procedure has been illustrated, which allows us to express a fermionic Gaussian system in terms of commuting variables by introducing an extra dimension. The Fermi–Bose duality principle established in this way also has many potential applications outside the context of gauge field theories in which it has been developed. In this work we present an application to the problem of averaging the correlation functions with respect to random potentials in disordered systems and similar problems.

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1. Introduction

Both in statistical mechanics and in quantum mechanics there are several situations in which one has to average the correlation functions of a physical system with respect to disorder fields [2]. The averaging procedure is complicated due to the fact that the dependence of the correlation functions on the disorder is not explicitly known. If we restrict ourselves to systems which admit a representation of the Green functions in terms of Gaussian fields, we will have three powerful tools at our disposal to solve this problem: the replica method [3], the supersymmetric method [4] and the so-called Keldysh approach [5, 6]. The replica method can be applied to more general cases than the supersymmetric method, but it has the disadvantage that its mathematical consistency has not yet been proved, so it has been subjected sometimes to some critiques concerning its validity in nonperturbative calculations [7]. On the other hand, the introduction of supersymmetric fields requires a fermionization of the system in which the passage from bosonic to fermionic degrees of freedom is often obscure. For this reason, difficulties arise for instance in systems with spontaneous symmetry breaking, because it is not easy to interpret the symmetry breaking in terms of the resulting fermionic theory [8]. Moreover, fermions are particularly difficult to treat numerically. Finally, the Keldysh technique requires the introduction of an extra dimension and a time ordering, but it is a very powerful technique and also allows the treatment of systems out of equilibrium.

In this letter we propose an alternative to the above methods. Our approach is very similar in spirit to the supersymmetric one and differs from the latter due to the fact that, instead of fermionization, we exploit the Fermi–Bose duality principle recently established by Slavnov [1] in the context of lattice gauge field theories in order to provide an expression of fermion determinants which does not contain anticommuting fields. Successively, this principle has also been applied in [9] to rewrite the Faddeev–Popov determinants without anticommuting ghosts. Since in our case all operators are Hermitian, it is possible to simplify the original procedure, which otherwise would have lead to field theories with derivatives of the fourth order in the action. Another relevant change with respect to [1, 9] is the choice of boundary conditions of the auxiliary bosonic fields. Here boundary conditions are dictated by the compatibility requirement with the regularization needed in the path integral approach to quantum mechanics to guarantee convergence.

The material presented in this work is organized as follows. In section 2, the problem of averaging over the disorder fields is briefly discussed. In section 3, our alternative method based on bosonization is presented. Conclusions are drawn in section 4.

2. The averaging problem in disordered systems

Let H be a local and Hermitian Hamiltonian describing a system with D degrees of freedom $\mathbf{q} = (q_1, \ldots, q_D)$. Further, we suppose that H depends on a set of random potentials $\vec{\varphi}(\mathbf{q}) = (\varphi_1(\mathbf{q}), \varphi_2(\mathbf{q}), \ldots)$ with a given distribution $P(\vec{\varphi})$. Hamiltonians of this kind are widely applied in quantum mechanics [2]. For instance, choosing $\vec{\varphi} = (\varphi_1)$, one obtains the energy operator of a disordered system

$$H = H_0 + \varphi_1 \tag{1}$$

consisting of a fixed Hamiltonian H_0 and a random perturbation $\varphi_1(\mathbf{q})$. If, for example, φ_1 is a source of Gaussian noise, the general form of $P(\varphi_1)$ is given by

$$P(\varphi_1) = \exp\left\{-\int \mathrm{d}^D q\, \mathrm{d}^D q'\, \varphi_1(\mathbf{q}) K(\mathbf{q}, \mathbf{q}')\varphi_1(\mathbf{q}')\right\}. \tag{2}$$

Analogously, with the same formalism it is possible to discuss the motion of n-dimensional particles immersed in an electromagnetic field with components A_i, $i = 1, \ldots, n$, putting $\vec{\varphi} = (A_1, \ldots, A_D)$ and taking as 'distribution'

$$P(A_1, \ldots, A_D) = \exp\{-\mathrm{i}S_{\mathrm{QED}}\} \tag{3}$$

where

$$S_{\mathrm{QED}} = \frac{1}{4g}\int \mathrm{d}^D x\, F_{ij}^2 \tag{4}$$

is the usual action of quantum electrodynamics in n dimensions and $F_{ij} = \partial_i A_j - \partial_j A_i$.[1]

In the following, we denote the quantum average and the average over disorder fields with the symbols $\langle\quad\rangle$ and $(\quad)_{\vec{\varphi}}$, respectively. With this notation, the advanced and retarded Green functions of the Hamiltonian H are given by

$$G_E^{\pm}(\mathbf{q}, \mathbf{q}'; \vec{\varphi}) = \lim_{\epsilon\to 0^+}\left\langle \mathbf{q} \left| \frac{1}{E \pm \mathrm{i}\epsilon - H} \right| \mathbf{q}' \right\rangle \tag{5}$$

[1] A situation in which the electromagnetic potentials can be treated as random potentials occurs for instance in polymer physics (see, e.g., [10]).

where $E + \mathrm{i}\epsilon$ is a complex parameter with an arbitrary small imaginary part $\mathrm{i}\epsilon$. Relevant information about the system may be obtained by computing averages over $\vec{\varphi}$ of products of the above Green functions:

$$\left(G_E^{\pm}(\mathbf{q},\mathbf{q}';\vec{\varphi})G_{E'}^{\pm}(\mathbf{q},\mathbf{q}';\vec{\varphi})\cdots\right)_{\vec{\varphi}} = \int \mathcal{D}\vec{\varphi}P(\vec{\varphi})G_E^{\pm}(\mathbf{q},\mathbf{q}';\vec{\varphi})G_{E'}^{\pm}(\mathbf{q},\mathbf{q}';\vec{\varphi})\cdots. \tag{6}$$

For this purpose, it is often convenient to use a representation of $G_E^{\pm}(\mathbf{q},\mathbf{q}';\vec{\varphi})$ in terms of complex scalar fields $\phi,\bar{\varphi}$:

$$G_E^{\pm}(\mathbf{q},\mathbf{q}';\vec{\varphi}) = \frac{1}{\mathrm{i}Z_{\pm}}\int \mathcal{D}\phi\mathcal{D}\bar{\phi}\phi(\mathbf{q})\bar{\phi}(\mathbf{q}')\exp\left\{\pm\mathrm{i}\int \mathrm{d}^D\mathbf{q}\bar{\phi}(E\pm\mathrm{i}\epsilon - H)\phi\right\}. \tag{7}$$

$Z_{\pm}$ represents the partition function of the field theory:

$$Z_{\pm} = \int \mathcal{D}\phi\mathcal{D}\bar{\phi}\exp\left\{\pm\mathrm{i}\int \mathrm{d}^D\mathbf{q}\bar{\phi}(E\pm\mathrm{i}\epsilon - H)\phi\right\}. \tag{8}$$

It is easy to realize that, even in the simple case of a single noise source with Gaussian distribution as in equation (2), it is difficult to integrate over the random potentials on the right-hand side of equation (6) due to the presence of the factor $Z_{\pm}^{-1}$ in the definition of $G^{\pm}(\mathbf{q},\mathbf{q}', E|\vec{\varphi})$ (see equation (7)). In fact, the partition function $Z_{\pm}$ is a functional depending on $\vec{\varphi}$ in a complicated way. In the following section, the possibility of performing the average with respect to the random potentials without introducing replica fields or fermionic degrees of freedom is shown.

3. The bosonization method

First of all, we note that the class of problems under investigation has a Gaussian nature, as is shown by the field theory representation of equation (7), in which only Gaussian fields are involved. Due to this fact, it will be sufficient to consider only the average of a single Green function for our aims. Let us study for instance the following average:

$$\langle G_E^{-}(\mathbf{q},\mathbf{q}';\vec{\varphi})\rangle_{\vec{\varphi}} = \int \mathcal{D}\vec{\varphi}P(\vec{\varphi})G_E^{-}(\mathbf{q},\mathbf{q}';\vec{\varphi}). \tag{9}$$

Now it will be convenient to interpret the factor $Z_{\pm}^{-1}$ in equation (7) as the functional determinant of the operator $E - H$:

$$Z_{\pm}^{-1} = \det(E \pm \mathrm{i}\epsilon - H). \tag{10}$$

To express the determinant appearing on the right-hand side of equation (10), we apply the Fermi–Bose duality principle proposed in [1]. For this purpose, we introduce a fictitious time τ such that

$$-T \leqslant \tau \leqslant T \tag{11}$$

and two sets of auxiliary fields $c_n(\mathbf{q},\tau)$, $\bar{c}_n(\mathbf{q},\tau)$ and $\chi_n(\mathbf{q})$, $n - 1, 2$. The field $\bar{c}_n$ is the Hermitian conjugate of c_n

$$\bar{c}_n = (c_n)^{\dagger} \tag{12}$$

while χ_n is a Hermitian scalar field, i.e. $(\chi_n)^{\dagger} = \chi_n$. The fields c_n and $\bar{c}_n$ satisfy the boundary conditions

$$c_n(\mathbf{q},-T) = B_n(\mathbf{q}) \qquad \bar{c}_n(\mathbf{q},T) = B_n^{\dagger}(\mathbf{q}) \tag{13}$$

where $B_n(\mathbf{q})$ is an arbitrary function of $\mathbf{q}$.

We are now ready to prove the following formula:

$$Z_{-}^{-1} = \det(E + \mathrm{i}\epsilon - H) = \lim_{T\to+\infty} \mathcal{Z}_{c,1}\mathcal{Z}_{c,2} \tag{14}$$

where

$$\mathcal{Z}_{c,n} = \int \mathcal{D}c_n \mathcal{D}\bar{c}_n \mathcal{D}\chi_n \, \mathrm{e}^{\mathrm{i}S_n} \tag{15}$$

and

$$S_n = \int_{-T}^{T} \mathrm{d}\tau \int \mathrm{d}^D q \left[-\left(\frac{\mathrm{i}}{2}\frac{\partial \bar{c}_n}{\partial \tau} + (E - H)\bar{c}_n \right) c_n + \left(\frac{\mathrm{i}}{2}\frac{\partial c_n}{\partial \tau} - (E - H)c_n \right) \bar{c}_n + 2\mathrm{i}\epsilon c_n \bar{c}_n + \chi(\bar{c}_n + c_n) \right]. \tag{16}$$

The proof is as follows. Since the operator $E - H$ is Hermitian, it supports a complete system of orthonormal eigenfunctions ψ_α with eigenvalues λ_α:

$$(E - H)\psi_\alpha(\mathbf{q}) = \lambda_\alpha \psi_\alpha(\mathbf{q}). \tag{17}$$

Thus, it is possible to expand the fields c_n, $\bar{c}_n$ and χ_n in terms of the ψ_α:

$$c_n(\mathbf{q}, \tau) = \sum_\alpha c_n^\alpha(\tau)\psi_\alpha(\mathbf{q}) \tag{18}$$

$$\bar{c}_n(\mathbf{q}, \tau) = \sum_\alpha \bar{c}_n^\alpha(\tau)\psi_\alpha(\mathbf{q}) \tag{19}$$

$$\chi_n(\mathbf{q}) = \sum_\alpha \chi_n^\alpha \psi_\alpha(\mathbf{q}). \tag{20}$$

Here $c_n^\alpha(\tau)$, $\bar{c}_n^\alpha(\tau)$ depend only on the pseudo-time τ, while the χ_n^α are constant coefficients. To these equations, one should add the expansions of the fields B_n and $B_n^\dagger$ which express the boundary conditions:

$$B_n(\mathbf{q}) = \sum_\alpha B_n^\alpha \psi_\alpha(\mathbf{q}) \qquad B_n^\dagger(\mathbf{q}) = \sum_\alpha \bar{B}_n^\alpha \psi_\alpha(\mathbf{q}). \tag{21}$$

Substituting equations (18)–(20) in the action (16) and recalling that

$$\int \mathrm{d}^D q \, \psi_\alpha(\mathbf{q})\psi_\beta(\mathbf{q}) = \delta_{\alpha\beta} \tag{22}$$

one obtains for $\mathcal{Z}_{c,n}$,

$$\mathcal{Z}_{c,n} = \prod_\alpha \int \mathcal{D}\bar{c}_n^\alpha \mathcal{D}c_n^\alpha \mathcal{D}\chi_n^\alpha \, \mathrm{e}^{\mathrm{i}S_{n,\alpha}} \tag{23}$$

with

$$S_{n,\alpha} = \int_{-T}^{T} \mathrm{d}\tau \left[-\left(\frac{\mathrm{i}}{2}\frac{\partial \bar{c}_n^\alpha}{\partial \tau} + \lambda_\alpha \bar{c}_n^\alpha \right) c_n^\alpha + \left(\frac{\mathrm{i}}{2}\frac{\partial c_n^\alpha}{\partial \tau} - \lambda_\alpha c_n^\alpha \right) \bar{c}_n^\alpha + 2\mathrm{i}\epsilon c_n^\alpha \bar{c}_n^\alpha + \chi_n^\alpha \left(\bar{c}_n^\alpha + c_n^\alpha \right) \right]. \tag{24}$$

The Gaussian path integral over the fields $\bar{c}_n^\alpha$ and c_n^α may be easily computed with the saddle point method. For this purpose, one has to solve the classical equations of motion of these fields:

$$\dot{c}_n^\alpha + \omega_\alpha c_n^\alpha - \mathrm{i}\chi_n^\alpha = 0 \tag{25}$$

$$\dot{\bar{c}}_n^\alpha - \omega_\alpha c_n^\alpha + \mathrm{i}\chi_n^\alpha = 0. \tag{26}$$

In the above equations we have put $\dot{c}_n^\alpha = \mathrm{d}c_n^\alpha/\mathrm{d}\tau$, $\dot{\bar{c}}_n^\alpha = \mathrm{d}\bar{c}_n^\alpha/\mathrm{d}\tau$ and

$$\omega_\alpha = 2(\mathrm{i}\lambda_\alpha + \epsilon). \tag{27}$$

The solutions of (25), (26) satisfying the desired boundary conditions are

$$c_{n,cl}^\alpha(\tau) = \mathrm{e}^{-\omega_\alpha(\tau+T)} B_n^\alpha + \mathrm{i}\frac{\chi_n^\alpha}{\omega_\alpha}[1 - \mathrm{e}^{-\omega_\alpha(\tau+T)}] \tag{28}$$

$$\bar{c}_{n,cl}^\alpha(\tau) = \mathrm{e}^{\omega_\alpha(\tau-T)} \bar{B}_n^\alpha + \mathrm{i}\frac{\chi_n^\alpha}{\omega_\alpha}[1 - \mathrm{e}^{\omega_\alpha(\tau-T)}]. \tag{29}$$

Let us note that $c_{n,cl}^\alpha$ and $\bar{c}_{n,cl}^\alpha$ do not diverge for large values of τ. Moreover, it is clear that the boundary conditions are irrelevant in the limit $T \to +\infty$, because their contribution vanishes exponentially as $\mathrm{e}^{-2\epsilon T}$.

After the field transformation

$$c_n^\alpha(\tau) = c_{n,cl}^\alpha(\tau) + c_{n,q}^\alpha(\tau) \tag{30}$$

$$\bar{c}_n^\alpha(\tau) = \bar{c}_{n,cl}^\alpha(\tau) + \bar{c}_{n,q}^\alpha(\tau) \tag{31}$$

$\mathcal{Z}_{c,n}$ becomes

$$\mathcal{Z}_{c,n} = \lim_{T\to+\infty} \prod_\alpha \int \mathcal{D}\chi_n^\alpha \mathcal{N}_{n,\alpha}\, \mathrm{e}^{\mathrm{i}S_{n,\alpha}^{cl}}. \tag{32}$$

Here we have put

$$S_{n,\alpha}^{cl} = \frac{1}{2}\int_{-T}^{T} \mathrm{d}\tau \left[2\mathrm{i}\frac{(\chi_n^\alpha)^2}{\omega_\alpha} - \mathrm{i}\frac{(\chi_n^\alpha)^2}{\omega_\alpha}\, \mathrm{e}^{-\omega_\alpha T}(\mathrm{e}^{-\omega_\alpha\tau} + \mathrm{e}^{\omega_\alpha\tau}) + \mathrm{i}\chi_n^\alpha\, \mathrm{e}^{-\omega_\alpha T}\left(\mathrm{e}^{-\omega_\alpha\tau} B_n^\alpha + \mathrm{e}^{\omega_\alpha\tau}\bar{B}_n^\alpha\right)\right] \tag{33}$$

and

$$\mathcal{N}_{n,\alpha} = \int \mathcal{D}c_{n,q}^\alpha \mathcal{D}\bar{c}_{n,q}^\alpha \exp\left[\mathrm{i}\int_{-T}^{T}\left(-\frac{\mathrm{i}}{2}\dot{\bar{c}}_{n,q}^\alpha c_{n,q}^\alpha + \frac{\mathrm{i}}{2}\dot{c}_{n,q}^\alpha \bar{c}_{n,q}^\alpha\right)\right]. \tag{34}$$

It is easy to show that the constant factor $\mathcal{N}_{n,\alpha}$ produced by the integration over the 'quantum' fields $c_{n,q}^\alpha$, $\bar{c}_{n,q}^\alpha$ is just a constant, which is independent of ω_α and thus can be ignored[2]. At this point it is possible to perform the integration over the pseudo-time τ in the action $S_{n,\alpha}^{cl}$. The result is

$$S_{n,\alpha}^{cl} = 2\mathrm{i}\frac{(\chi_n^\alpha)^2 T}{\omega_\alpha} - \mathrm{i}\frac{(\chi_n^\alpha)^2}{\omega_\alpha^2}(1 - \mathrm{e}^{-2\omega_\alpha T}) + \frac{\chi_n^\alpha}{\omega_\alpha}(1 - \mathrm{e}^{-2\omega_\alpha T})\left(B_n^\alpha + \bar{B}_n^\alpha\right). \tag{35}$$

Finally, one has to integrate over the variables χ_n^α in $\mathcal{Z}_{c,n}$:

$$\mathcal{Z}_{c,n} = \int \mathcal{D}\chi_n^\alpha \exp\left\{\mathrm{i}\left[2\mathrm{i}\frac{(\chi_n^\alpha)^2 T}{\omega_\alpha} - \mathrm{i}\frac{(\chi_n^\alpha)^2}{\omega_\alpha^2}(1 - \mathrm{e}^{-2\omega_\alpha T}) + \frac{\chi_n^\alpha}{\omega_\alpha}(1 - \mathrm{e}^{-2\omega_\alpha T})\left(B_n^\alpha + \bar{B}_n^\alpha\right)\right]\right\}. \tag{36}$$

Only the first term on the right-hand side of the above equation becomes relevant when T becomes very large, as in our case. Since

$$\frac{2\mathrm{i}T}{\omega_\alpha} = T\frac{\lambda_\alpha + \mathrm{i}\epsilon}{\lambda_\alpha^2 + \epsilon^2} \tag{37}$$

[2] Let us note that this factor is also independent of T.

it turns out that, thanks to the presence of the ϵ-term in the action S_n of equation (16), the integrals in $\mathcal{D}\chi_n^\alpha$ are convergent. Upon renormalizing the fields χ_n^α in equation (36) as follows: $\chi_n^{\alpha\prime} = \chi_n^\alpha T^{1/2}$, one finds

$$\mathcal{Z}_{c,n} = \prod_\alpha \sqrt{\lambda_\alpha + \mathrm{i}\epsilon} = \sqrt{\det(E - H + \mathrm{i}\epsilon)}. \tag{38}$$

This proves equation (14) as desired. An analogous formula can be derived for Z_+^{-1}.

Coming back to the original averaging problem, we rewrite equation (9) in the form

$$\langle G_E^-(\mathbf{q}, \mathbf{q}'; \vec{\varphi})\rangle_{\vec{\varphi}} = \lim_{T\to+\infty} \int \mathcal{D}\vec{\varphi} P(\vec{\varphi}) \mathcal{Z}_{c,1}\mathcal{Z}_{c,2} \int \mathcal{D}\phi\mathcal{D}\bar{\phi}\phi(\mathbf{q})\bar{\phi}(\mathbf{q}')$$
$$\times \exp\left\{-\mathrm{i}\int_{-T}^{T} \frac{\mathrm{d}\tau}{2T} \int \mathrm{d}^D\mathbf{q}\bar{\phi}(E - \mathrm{i}\epsilon - H)\phi\right\}. \tag{39}$$

The dependence of $G_E^-(\mathbf{q}, \mathbf{q}'; \vec{\varphi})$ on the disorder fields $\vec{\varphi}$ is now explicit and is given by equation (16), which expresses the inverse of the partition function Z_- as a path integral over Gaussian fields $c_n, \bar{c}_n, \chi_n, n = 1, 2$. At this point it is possible to perform the averaging over the disorder fields at least perturbatively. In the case of Hamiltonians such as that of equation (1), the random potential φ_1 may be integrated from the partition function with the help of a Gaussian integral.

Let us note that in equation (39) the limit for $T \to +\infty$ has been permuted with the integration over the disorder fields $\vec{\varphi}$. In a similar way, in the method of replicas one needs to permute the limit of vanishing replicas and the average with respect to the disorder. The difference between the two approaches is that in the present case the limit $T \to +\infty$ does not require a complex analytical continuation as in the method of replicas and it is mathematically under control. In fact, the presence of the variable T is only limited to the partition functions $\mathcal{Z}_{c,n}$ of the bosonic fields $c_n, \bar{c}_n, \chi_n$. These partition functions may always be rewritten as in equation (36), i.e. in terms of standard integrals over the real variables χ_n^α, which are convergent in the limit $T \to +\infty$ due to the presence of the ϵ-term. As a consequence, the permutation of the symbol $\lim_{T\to+\infty}$ with the integrals which are necessary to compute $\mathcal{Z}_{c,n}$ is allowed.

4. Conclusions

In conclusion, in this work a bosonization approach to disordered systems based on the Fermi–Bose principle of [1] has been presented. As in the Keldysh approach, bosonization also requires an extra dimension. The difference is that in the present case three distinct pairs of complex scalar fields $\bar{\phi}(\mathbf{q})$, $\phi(\mathbf{q})$ and $c_n(\mathbf{q}, \tau)$, $\bar{c}_n(\mathbf{q}, \tau)$, $n = 1, 2$, are required, while in the Keldysh approach there are just two pairs. Moreover, in the bosonization approach not all the scalar fields depend on the pseudo-time τ as in the Keldysh approach. The reason is that, as in the supersymmetric and replica methods, our treatment considers only the factor $Z_\pm^{-1}$, which makes the integration over the disorder fields difficult. The Keldysh method applies instead to the whole Green function $G_E^\pm(\mathbf{q}, \mathbf{q}'; \vec{\varphi})$ of equation (7). Despite these differences, there are also some analogies. It would thus be really nice if the bosonization method presented here could be used to interpolate between the supersymmetric and Keldysh methods showing their equivalence explicitly, but so far the feasibility of this programme is still an open question.

Since the bosonization approach does not contain fermionic degrees of freedom, it may be useful in investigating systems with spontaneous symmetry breaking, which are sometimes complicated by the presence of anticommuting variables. Let us also note that in the case of quantum chromodynamics the Fermi–Bose principle used to express the Faddeev–Popov determinant without the help of ghost fields leads to a new symmetry, which replaces the usual BRST symmetry. Therefore, it would be interesting to check if the disordered path integral of equation (39) also enjoys an analogous new symmetry, which would replace the fermion–boson symmetry of the supersymmetric method. Finally, as always happens in the case of a new approach, several natural questions arise. For instance, an interesting question is if the bosonization approach can be used to study non-perturbative effects in disordered systems. In principle, the answer is yes, in the sense that the Fermi–Bose principle of [1], which has been exploited here, has been developed having in mind non-perturbative numerical simulations. Concerning analytical calculations starting from concrete models, this work is still in progress.

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